DECIDABILITY OF TIMED COMMUNICATING AUTOMATA

by L. Clemente

Definations:

A Communication Topology is a directed graph $\langle P, C \rangle$ Where P is finite set of Processes, $C \subseteq P^2$

$pq \in C$ represents a channel from p to q

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A system of Timed Communi-cating Automata is a tuple

$$S := \langle \mathcal{T}, M, (X^c)_{c \in C}, (A^p)_{p \in P} \rangle$$

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where

A^p is a Timed Automata

for each process p

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Where

L^P: Set of locations

$l_I^P, l_F^P \in L^P$ initial & Final locations

X^P : Local clocks

Δ^P : $l_1^P \xrightarrow{op} l_2^P$

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- Send($pq, m : \psi$) Sends m to q

  ψ over $X^p \cup X^{pq}$ specifies the initial values of X^{pq}
- receive($qp, m : \psi$) recieves m from q

  ψ over $X^p \cup X^{qp}$ specifies the final value

Definations:

Clocks: Classical

Integral

Fractional

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Guards:

	ineq.	modular	order
diag:	$x_0 \leq k$	$x_0 \equiv_m k$	$y_0 = 0$
non-diag:	$x_0 - x_1 \leq k$	$x_0 - x_1 \equiv_m k$	$y_0 \leq y_1$

$m \in \mathbb{N}$ $\quad k \in \mathbb{Z}$

Definations:

A channel $c \in C$ has inequality test if there exist atleast one op. Send $(c, m; \psi)$ or Recive $(c, m : \psi)$ s.t. ψ is inequality test

(over classical or integral clocks)

Definations:

A channel valuation is a family $\omega = (\omega^c)_{c \in C}$ of sequences $\omega^c \in (M \times \mathbb{R}_{\geq 0}^{X^c})^*$ of pairs (m, u) where m is the message and u is the channel clock valuation

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C^I: Initial Config

$$\langle (\ell_I^p)_{p \in P}, \bar{0}, (\epsilon)_{c \in C} \rangle$$

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μ: a local clock valuation

c_I: Initial Config

$\langle (\ell_I^p)_{p \in P}, \bar{0}, (\epsilon)_{c \in C} \rangle$

c_F: Final Config

$\langle (\ell_F^p)_{p \in P}, \bar{0}, (\epsilon)_{c \in C} \rangle$

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$$A = \bigcup_{p \in P} O_p^{\,p} \cup \mathbb{Q}_{\geq 0}$$

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Set of actions:

$$\langle (l^p)_p, \mu, u \rangle \xrightarrow{\delta,} \langle (t^p)_p, \vartheta, v \rangle$$

if for all process p, there is a elapse transition

$$l^p \xrightarrow{\text{elapse}} t^p$$

$$\vartheta = \mu + \delta \qquad v = u + \delta$$

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for $q \neq r$: $t^r = \ell^r$

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- nop: $\mu = \vartheta$, $u = v$
- test(φ): $\mu \models \varphi$, $\vartheta = \mu$, $v = u$

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- nop: $\mu = \vartheta$, $u = \upsilon$
- test(φ): $\mu \models \varphi$, $\vartheta = \mu$, $\upsilon = u$
- Send$(qp, m; \psi)$: $\vartheta = \mu$

$\exists\, \mu^{qp} \in \mathbb{R}_{\geq 0}^{X^{qp}}$ s.t. $\mu \cup \mu^{qp} \models \psi$

$\upsilon^{qp} = (m, \mu^{qp}) \cdot u^{qp}$

$\upsilon^{c} = u^{c}$ for $c \neq qp$

Definations:

Set of actions:

$$\langle (\ell^p)_p, \mu, u \rangle \xrightarrow[op \in Op^q]{} \langle (t^p)_p, \nu, v \rangle$$

- nop: $\mu = \nu$, $u = v$
- test(φ): $\mu \models \varphi$, $\nu = \mu$, $v = u$
- Send($qp, m; \psi$): there exist

$\mu^{qp} \in \mathbb{Q}_{\geq 0}^{X^{qp}}$ s.t. $\mu \cup \mu^{qp} \models \psi$

$v^{qp} = (m, \mu^{qp}) . u^{qp}$

$v^c = u^c$ for $c \neq qp$

- Receive($pq, m : \psi$): $\nu = \mu$

$u^{pq} = (m, \mu^{qp})\, v^{pq}$

$u^c = v^c \quad c \neq pq$

Definations:

The semantics of TCA is

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S is non-empty, if there exists an execution in the semantics with predefined initian and the final config.

Main Theorem:

Non-emptiness of TCA is decidable

$\Updownarrow$

The communication topology is a polyforest s.t. in each polytree, there is atmost one channel with ineq. test

Only If direction:

- If the topology is not poly forest, non-emptiness is undecidable in untimed settings [BZ '83, Pac '82]

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- If the topology is not poly forest, non-emptiness is undecidable in untimed settings [B-Z '83, Pac '82]

- Poly forest but more than one channel ineq. test in a tree

$\Rightarrow$ Non-emptiness undecidable

[C-H-S '13]

If direction:

A TCA S is Simple TCA if it contains only integral & fractional clocks.

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Send Constraints: $x^c = 0$

Receive Constraints:

int. clocks $\begin{cases} x^c \sim k \\ x^e \equiv_M k \end{cases}$

frac clocks $\{ \; y^{pq} = y^q$

If direction:

Lemma

Non-emptiness for TCA reduces to Non-emptiness for Simple TCA

More definitions 🥲

Desynchronised Semantics

of TCA S is

$$[\![S]\!]^{de} = \langle C^{de}, C_I, C_F^{de}, A, \rightarrow^{de} \rangle$$

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$(\forall p, \exists\, x_0^p \in X^p,$ s.t. x_0^p is never reset, never appears in any constraints$)$

More definitions 🥲

Desynchronised Semantics

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$$[\![S]\!]^{de} = \langle C^{de}, C_I, C_F^{de}, A, \rightarrow^{de} \rangle$$

$$C^{de} : \left\{ \langle (\ell^p)_{p \in P}, \mu, u \rangle \;\middle|\; \begin{array}{l} \forall pq \in C \\ \mu(x_0^p) \leq \mu(x_0^q) \end{array} \right\}$$

More definitions 🥲

Desynchronised Semantics of TCA S is

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$$C_F^{de} : \langle (\ell_F^p)_{p \in P}, \mu_1, (\varepsilon)_{c \in C} \rangle$$

$$\mu_1(x^p) = 0$$
$$\forall x^p \neq x_0^p$$

More definitions 🥲

Desynchronised Semantics

of TCA S is

$$[\![S]\!]^{de} = \langle C^{de}, C_I, C_F^{de}, A, \to^{de} \rangle$$

$Op = \text{Send}(pq, m : \psi)$

$\exists\, u^{pq} \in \mathbb{Q}_{\geq 0}^{X^{pq}}$ s.t.

$u, u^{pq} \models \psi \qquad v^{pq} = (m, u^{pq} + \delta)\, u^{pq}$

Where $\delta := u(x_0^q) - u(x_0^p) \geq 0$

More definitions 🥲

$[\![S]\!]$ and $[\![S]\!]^{de}$ are equivalent in terms of non-emptiness problem

[C-H-S '13]

More definitions 🥲

Rendezvous Semantics

of TCA S is

$$[\![S]\!]^{rv} \doteq \langle C^{rv}, C_I, C_F, A^{rv}, \rightarrow^{rv} \rangle$$

$$C^{rv} := \{ \langle (\ell^p)_{p \in P}, \mu, (\varepsilon^c)_{c \in C} \rangle \}$$

More definitions 🥲

Rendezvous Semantics

of TCA S is

$$[\![S]\!]^{rv} \doteq \langle C^{rv}, C_I, C_F, A^{rv}, \rightarrow^{rv} \rangle$$

Modifying Send & Receive op.

in to one rule:

$$\langle (\ell^p)_p, \mu, (\varepsilon)_c \rangle \xrightarrow{op^p,\, op^q} \langle (t^p)_p, \mu, (\varepsilon)_c \rangle$$

op^p : Send

op^q : Recieve

If direction:

Lemma:

Over Simple TCA S

$[\![S]\!]^{rv} \equiv [\![S]\!]^{de}$ in terms

of non-emptiness decidability

[H-L-M-S '12]

More definitions 🥲

Let R be set of registers taking values from

$$\mathbb{I} := \mathbb{Q} \cap [0,1)$$

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Cyclic order structure:

$$\mathcal{K} := (\mathbb{I}, K)$$

where $K \subseteq \mathbb{I}^3$ is strict ternary cyclic order on a, b, c

$$K(a,b,c) \equiv a < b < c \vee b < c < a \vee c < a < b$$

More definitions 🥲

Register constraint on R is set of first-order formulas over $\mathcal{X}$

More definitions 🥲

Register Aut. with counters

$R := \langle L, l_I, l_F, R, N, \Delta \rangle$

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with $n{+}{+}$ increment op.

$n{-}{-}$ decrement op.

$n \sim k$ ineq. test

$n \equiv_m k$ modular test

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Register Aut. with counters

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guess(r): assigning non-det. value to $r \in R$

test(φ): test register constraint φ

More definitions 🥲

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More definitions 🥲

$[\![R]\!] ::= \langle C, C_I, C_F, A, \rightarrow \rangle$

$C ::= \{ \langle \ell, n, r \rangle \}$

- ℓ: location
- n: Counter valuation $\in \mathbb{N}^N$
- r: Register valuation $\in \mathbb{I}^R$

More definitions 🥲

$[\![R]\!] := \langle C, C_I, C_F, A, \rightarrow \rangle$

$C := \{ \langle l, n, r \rangle \}$

$C_I = \langle l_I, \bar{0}, \bar{0} \rangle$

$C_F = \langle l_F, \bar{0}, \bar{0} \rangle$

More definitions 🥲

$[\![R]\!] := \langle C, C_I, C_F, A, \to \rangle$

We write $\langle \ell, n, r \rangle \xrightarrow{op} \langle t, m, s \rangle$

if there is a rule $\ell \xrightarrow{op} m$ s.t.

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- nop: $m = n$, $r = s$
- x++ : $m = n[n(x)+1]$, $r = s$
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- test$(x \sim k)$: $n(x) \sim k$, $m = n$, $r = s$

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- x++: $m = n[n(x)+1]$, $r = s$
- x--: $m = n[n(x)-1]$, $r = s$
- test $(x \sim k)$: $n(x) \sim k$, $m = n$, $r = s$
- guess (r): $m = n$, $\exists x \in \mathbb{I}$
  s.t. $r[r \rightarrow x]$
- test (φ): $r \models \varphi$, $r = s$, $m = n$

If direction:

Theorem:

Non-emptiness is decidable for RAC with ≤ 1 Counter with inequality tests

If direction:

We can simulate a Polytree Simple TCA S by RAC $\mathcal{R}$ s.t. $[\![\mathcal{R}]\!] \equiv [\![S]\!]^{r.v}$ in terms of decidability of non-emptiness problem

THANK

YOU